

CS61A

# section 2

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<http://links.cs61a.org/jasonxu>

## **upcoming**

lab 2

hw 2

hog

## **feedback**

<http://links.cs61a.org/jasonxu-feedback>

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# last week...

inputs of functions

internals of functions

outputs of functions

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# review

1.  $100 \% 10 =$

2.  $241241 // 10 =$

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# booleans

bools have rules

not > and > or

TRUE

FALSE

1

0

'non-empty' values

'empty' values

None

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# boolean operators

and      `<a> and <b> ...`

or        `<a> or <b> ...`

not      `not <a>`

a	b
1	0
1	1
0	0

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# boolean operators

and       $\langle a \rangle$  and  $\langle b \rangle$  ...

or         $\langle a \rangle$  or  $\langle b \rangle$  ...

not      not  $\langle a \rangle$

a	b
1	0
1	1
0	0

which 1?

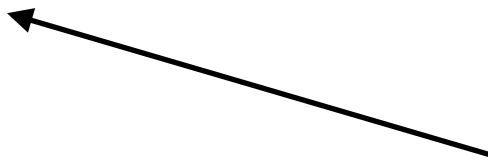
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# boolean operators another approach

if it's sunny and not hot

i will go for a run

only will do so when '<True> and <True>'



if it's sunny or not hot

i will go for a run

will do so when either condition is true



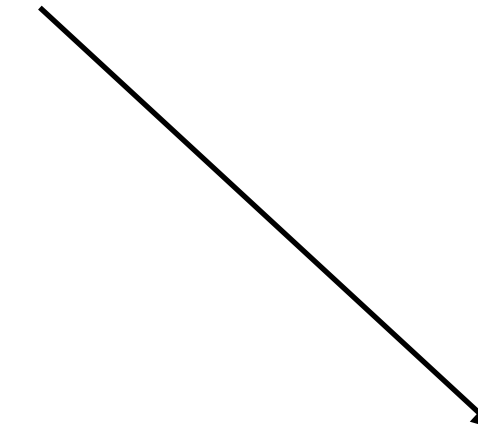
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# booleans

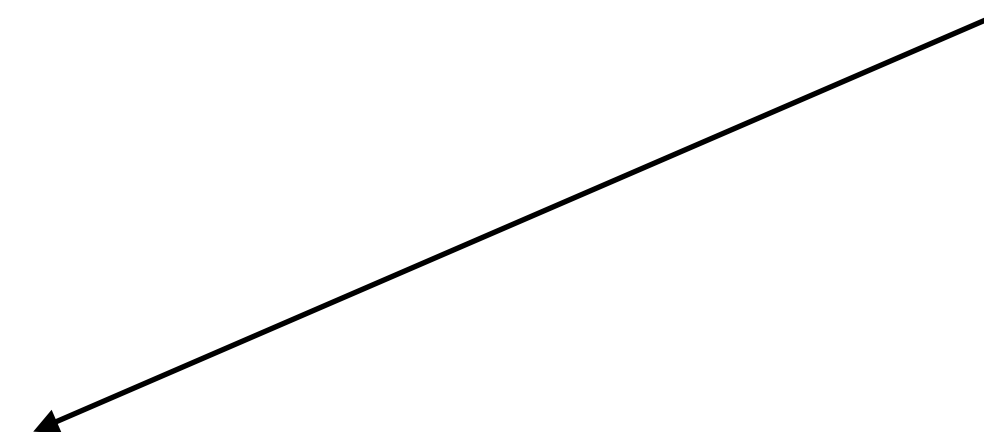
bools have rules

and looks for False

or looks for True



# short circuit!



we can process faster

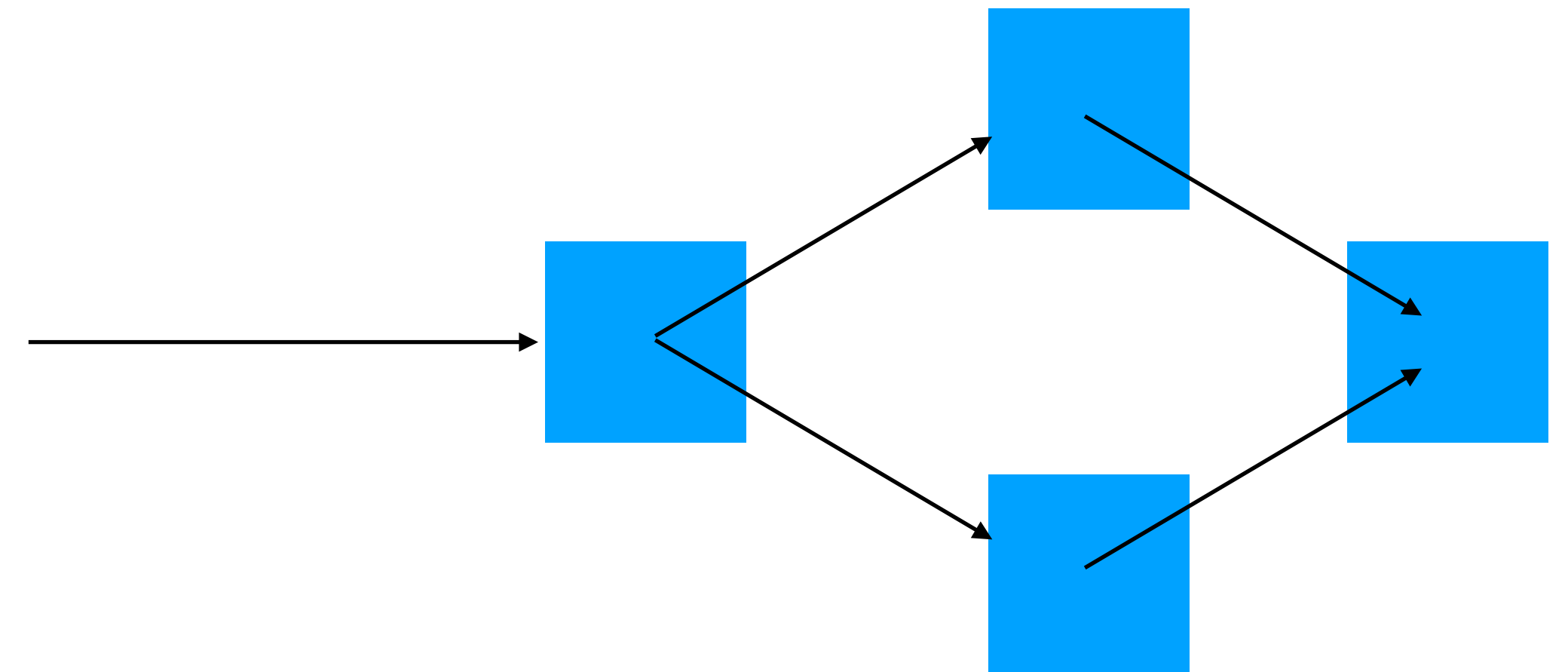
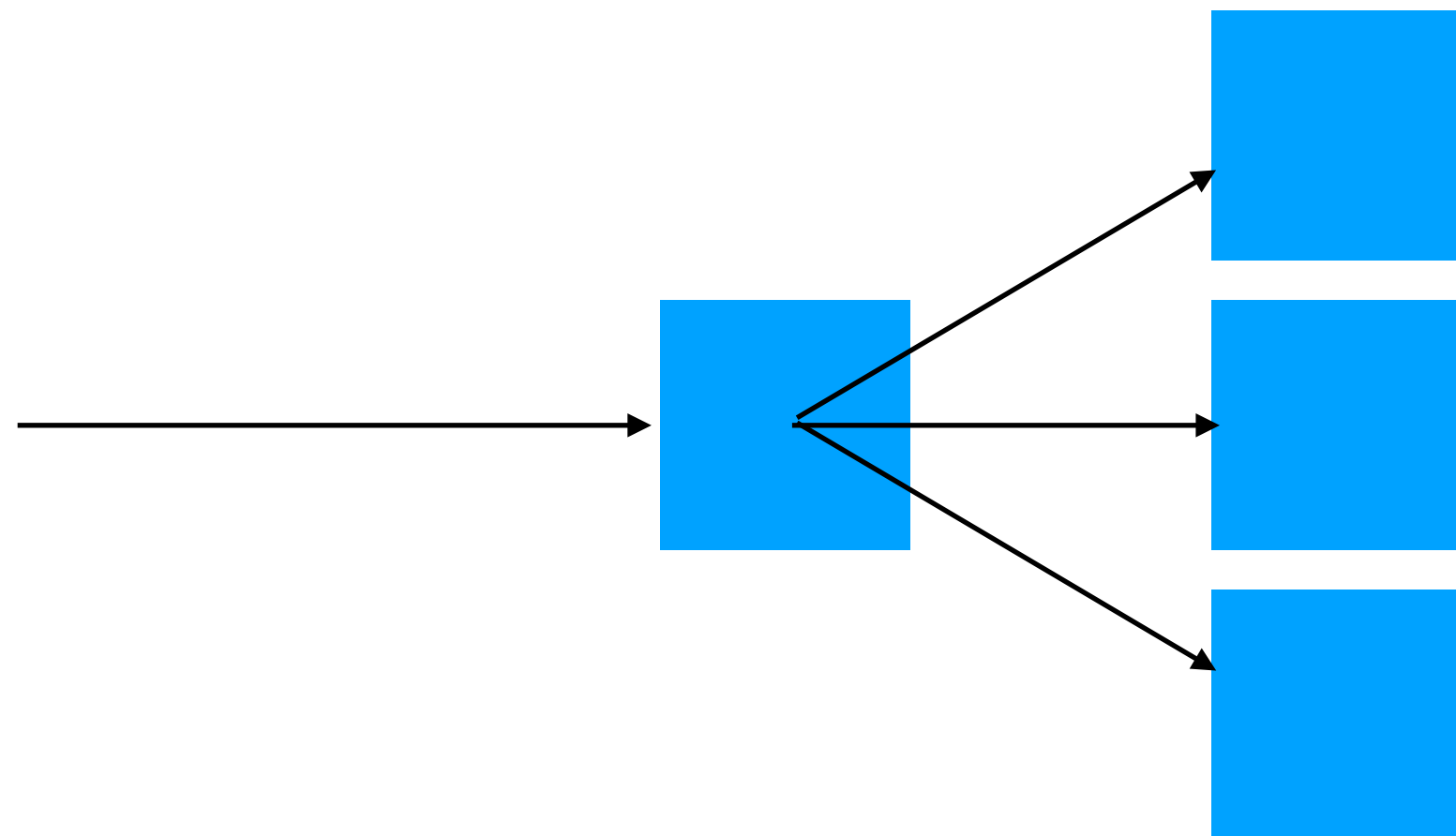


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# controls

```
if <predicate>:  
  <do this>  
elif <predicate>:  
  <do this>  
else:  
  <do this>
```

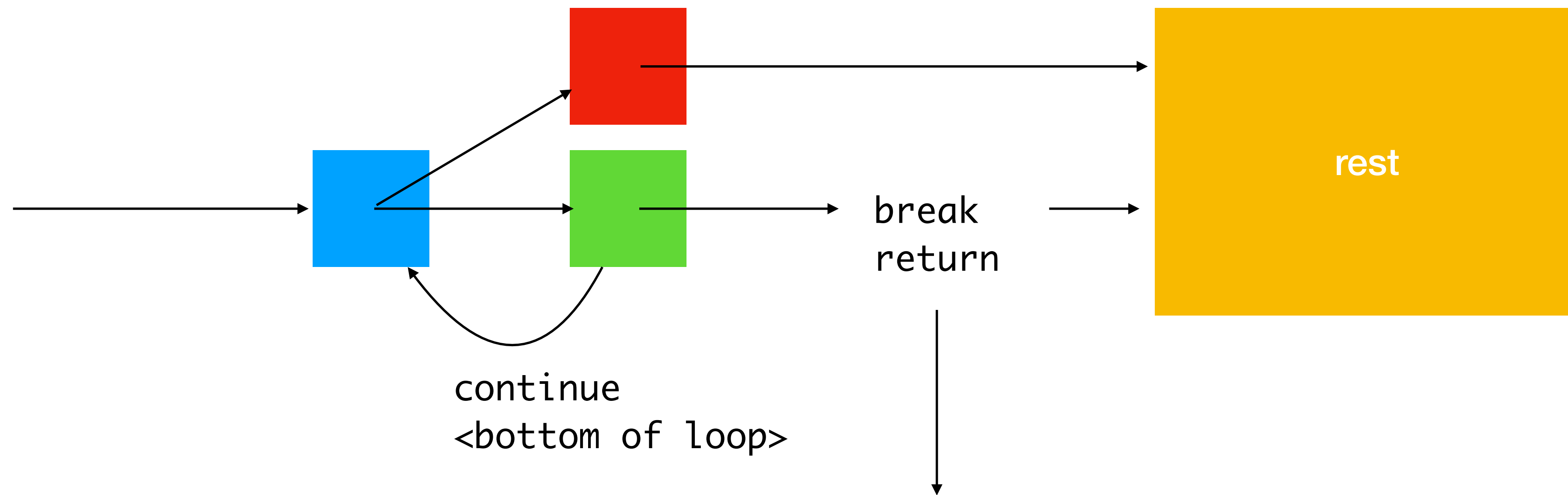
```
if <predicate>:  
  <do this>  
elif <predicate>:  
  <do this>  
<do this>
```



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# controls

```
while <this>:  
  <do this>  
<rest>
```



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# function calls

$$f(3 \times 2) =$$

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# function calls

$$f(3 \times 2) = ?$$

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# function calls

$f(x) = 2x$     general formula, i can put in any x that is a number

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# function calls

$f(x) = 2x$     general formula, i can put in any x that is a number

$$x = 3 \times 2$$

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# function calls

$f(x) = 2x$       general formula, i can put in any x that is a number

$x = 3 \times 2$

$f(x) = 12$       i know f, x, can solve!

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# function calls

$$f(x) = 2x$$

$$\textit{double}(z) = 2z$$

$$x = 3 \times 2$$

$$x = 3 \times 2$$

$$f(x) = 12$$

$$\textit{double}(x) = 12$$



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# function calls

$$f(x) = 2x$$

$$x = 3 \times 2$$

$$f(x) = 12$$

$$\textit{double}(z) = 2z$$

$$x = 3 \times 2$$

$$\textit{double}(x) = 12$$

$$\textit{double}(z) = 2z$$

$$\textit{double}(3 \times 2)$$

$$= 12$$

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# functions

There are 2 things to consider for a function

1. Input/Output of function

2. Body of function

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# functions

There are 2 things to consider for a function

1. Input/Output of function

2. Body of function

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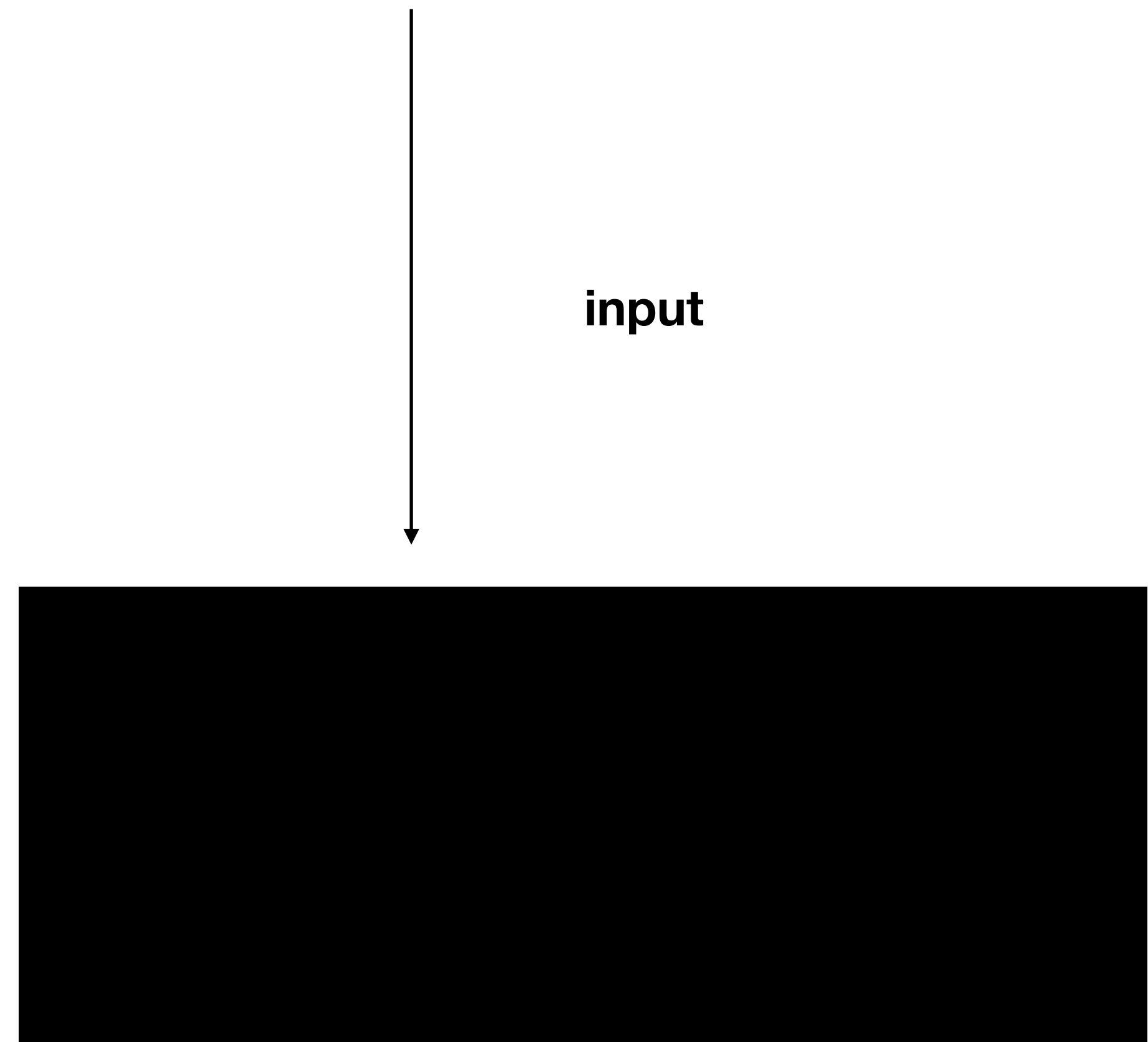
# black boxes

we're going to use these to see an abstract picture  
of functions

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# black boxes

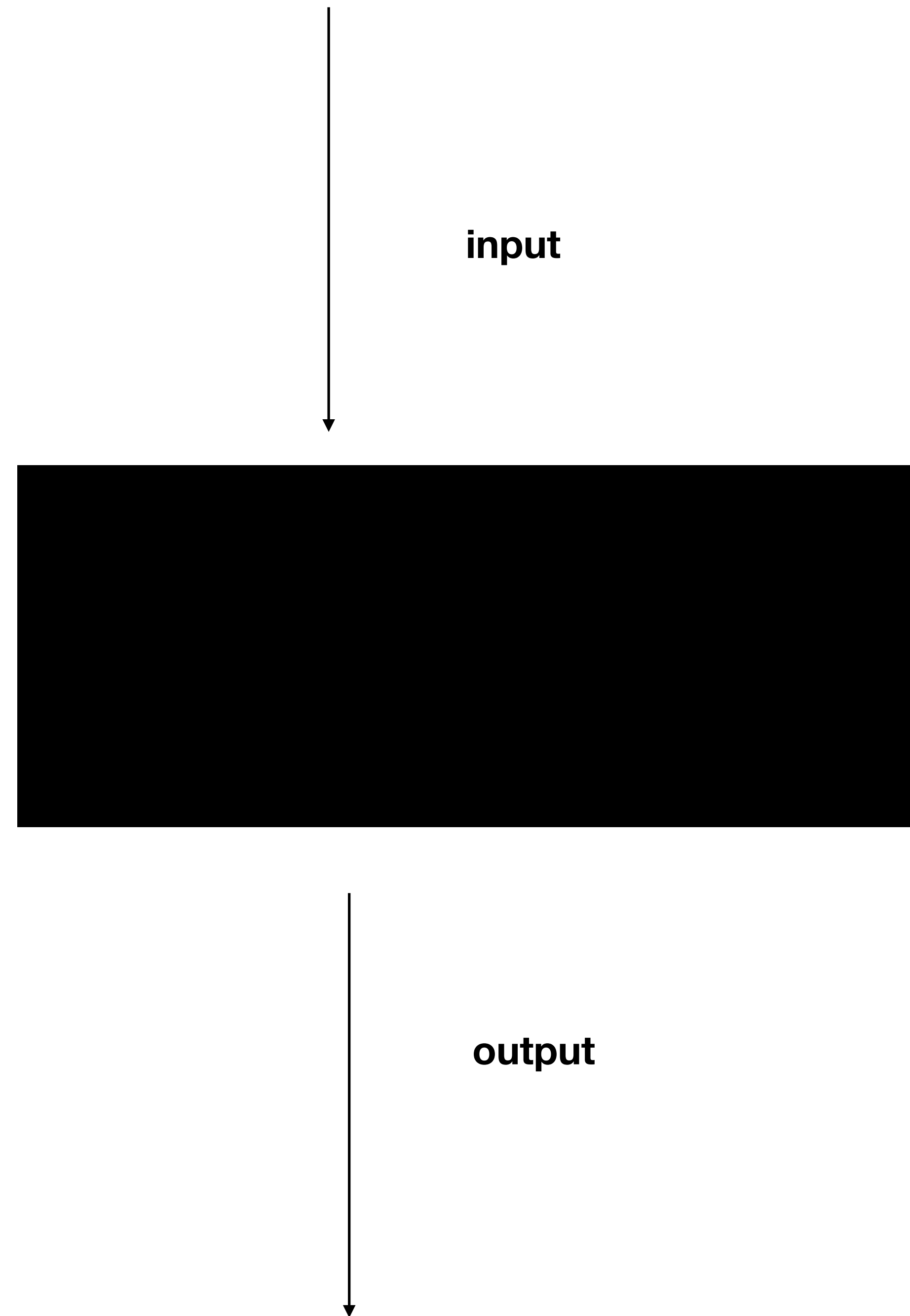
we're going to use these to see an abstract picture  
of functions



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# black boxes

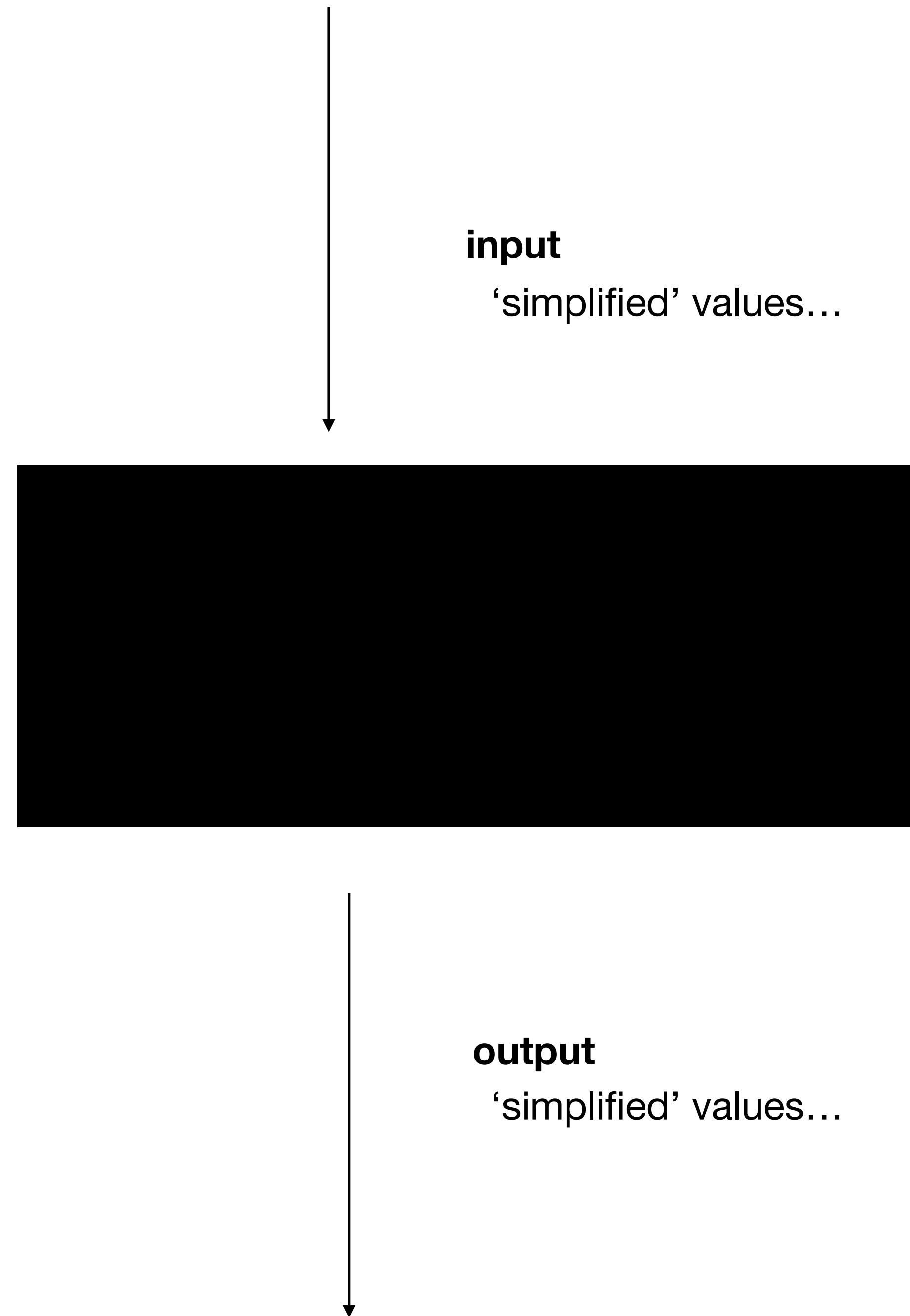
we're going to use these to see an abstract picture of functions



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# black boxes

we're going to use these to see an abstract picture  
of functions



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# values

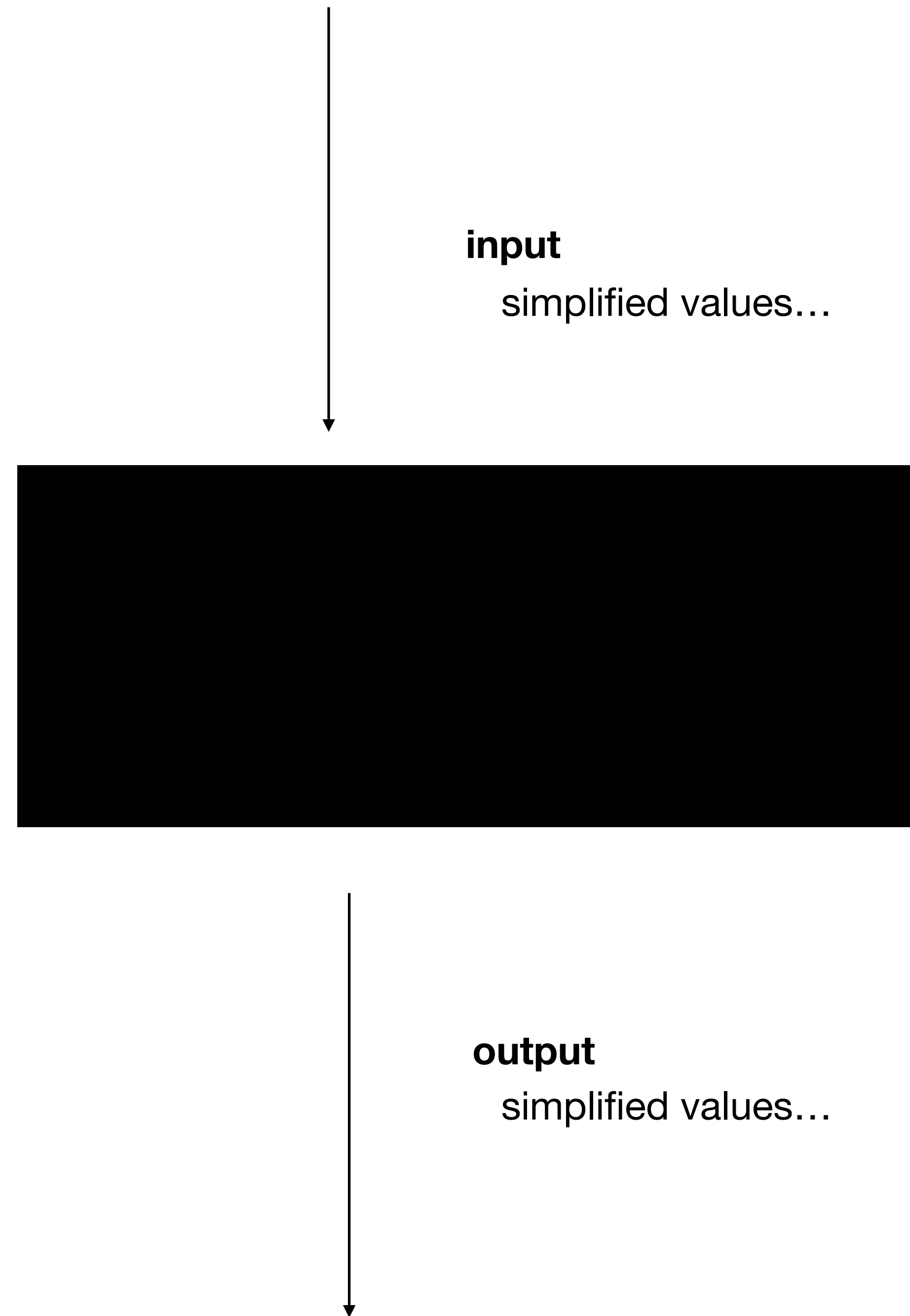
Type	Values	Literals (Denotations)
Integers	0 -1 16 13 36893488147419103232	0 -1 0o20 0b1101 0x20000000000000000000
Boolean (truth) values "Null"	true, false	True False None
Functions		operator.add, operator.mul, operator.lt, operator.eq
Strings	Say "Hello"	"Say \"Hello\""



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# black boxes

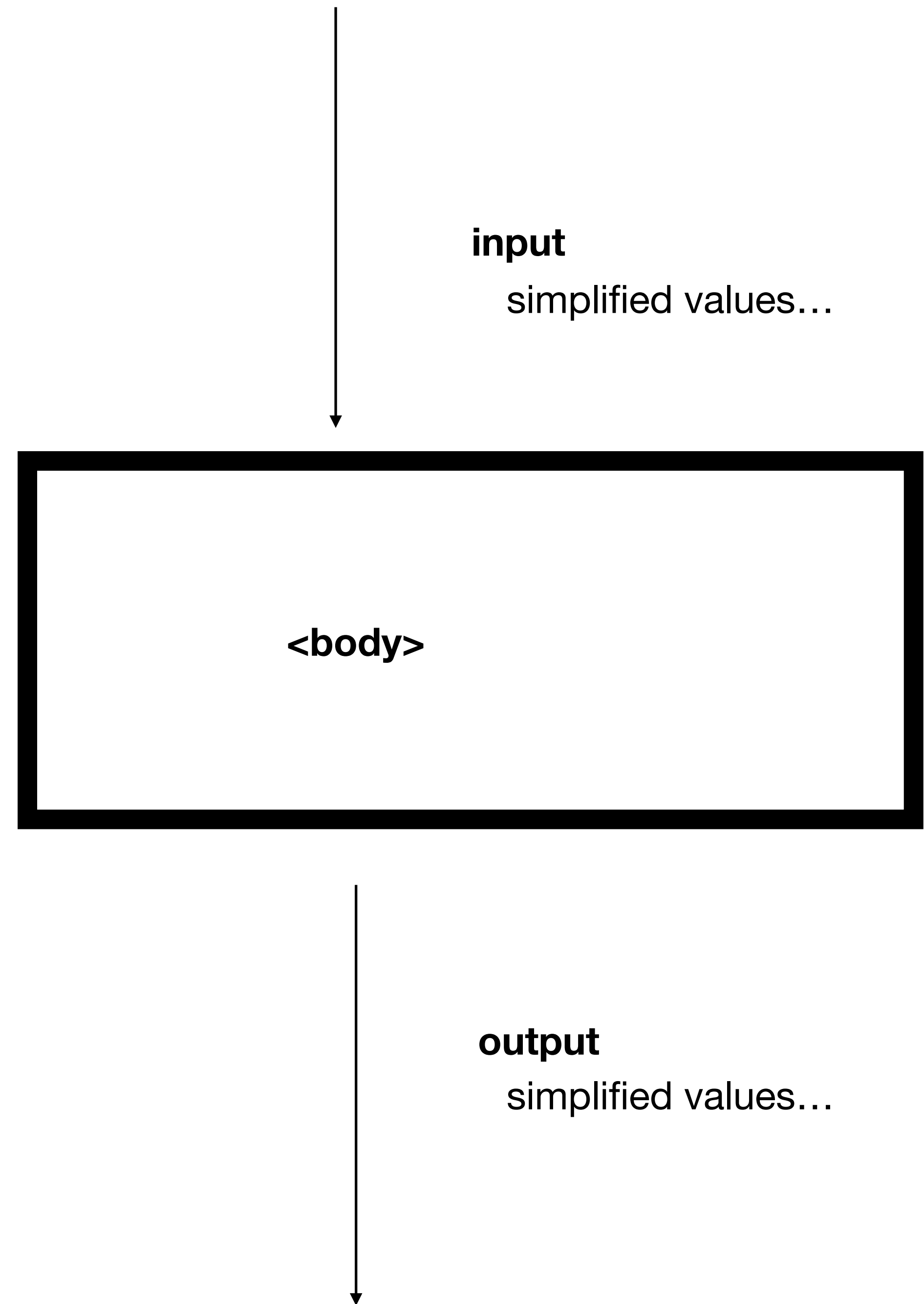
we're going to use these to see an abstract picture  
of functions



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# black boxes

we're going to use these to see an abstract picture of functions



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# returning

return stops procedure and outputs something

print is an action, function

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# returning

return stops procedure and outputs something

print is an action, function

```
def showFivePrint():  
    x = 2 + 3  
    print(x)
```

```
def showFiveReturn():  
    x = 2 + 3  
    return x
```

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# returning

return stops procedure and outputs something

print is an action, function

```
def showFivePrint():  
    x = 2 + 3  
    print(x)
```

```
def showFiveReturn():  
    x = 2 + 3  
    return x
```



computing 2 + 3 and storing it to x

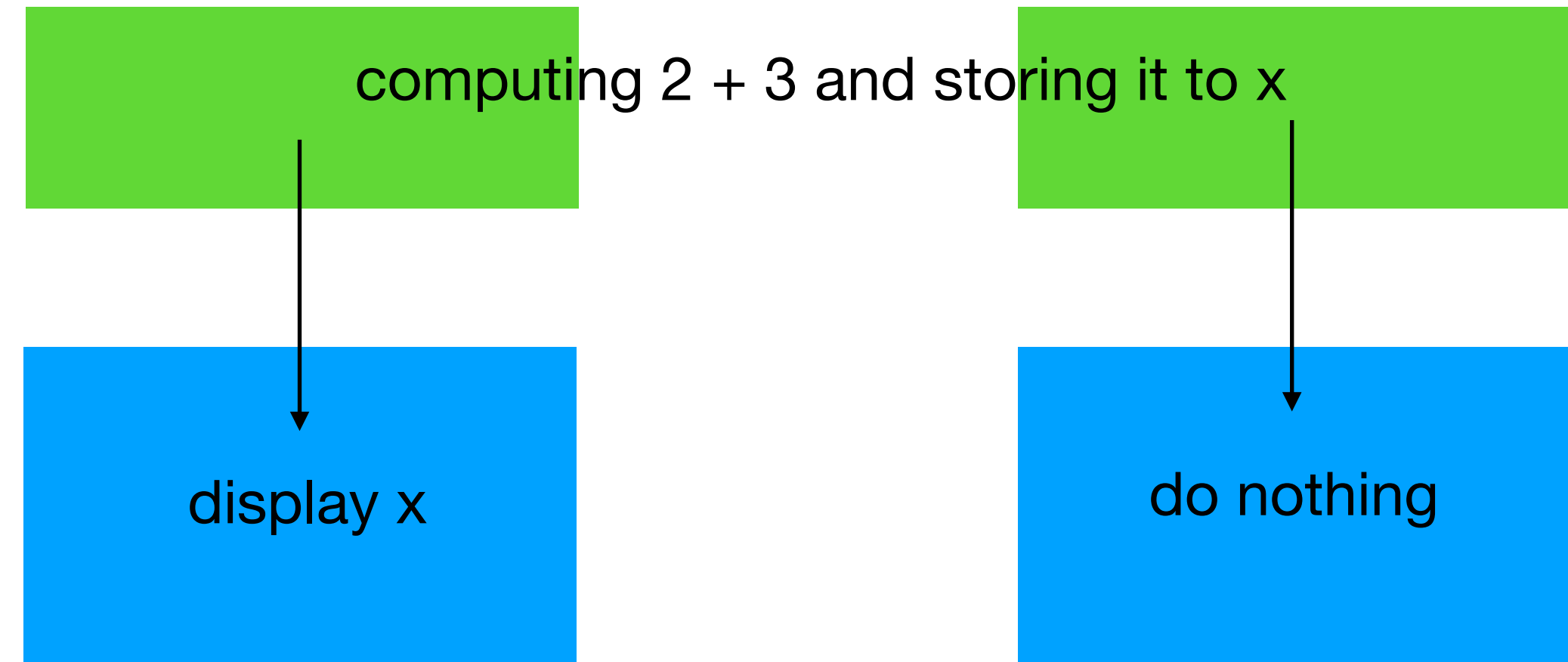
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# returning

return stops procedure and outputs something

print is an action, function

```
def showFivePrint():  
    x = 2 + 3  
    print(x)
```



```
def showFiveReturn():  
    x = 2 + 3  
    return x
```

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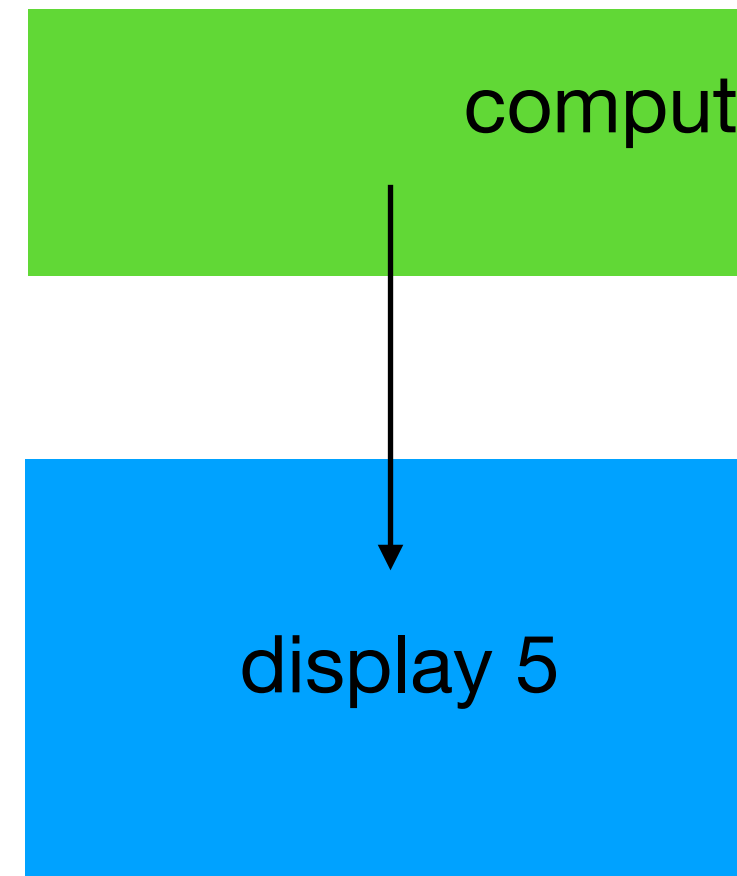
# returning

return stops procedure and outputs something

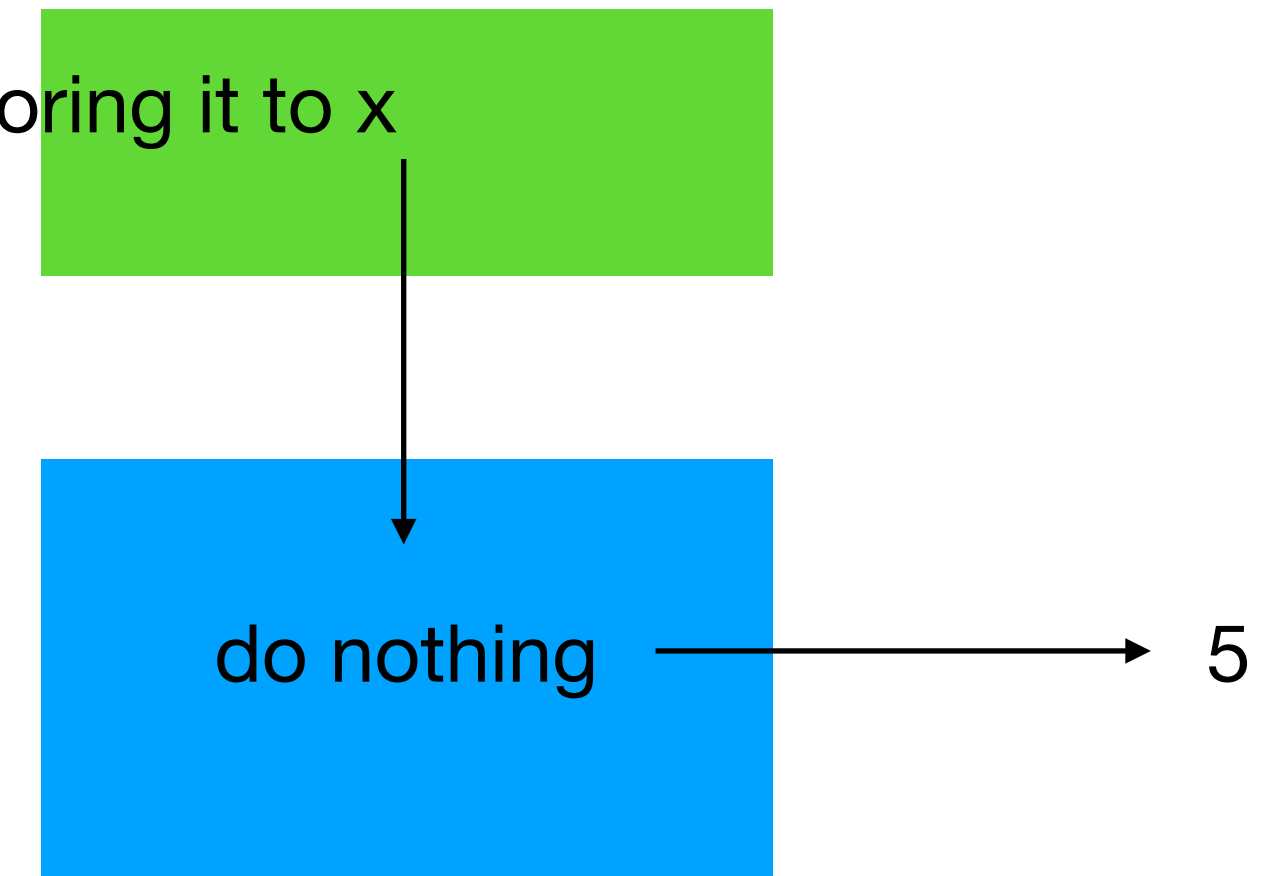
print is an action, function

every function has a return at end of None

```
def showFivePrint():  
    x = 2 + 3  
    print(x)
```



```
def showFiveReturn():  
    x = 2 + 3  
    return x
```

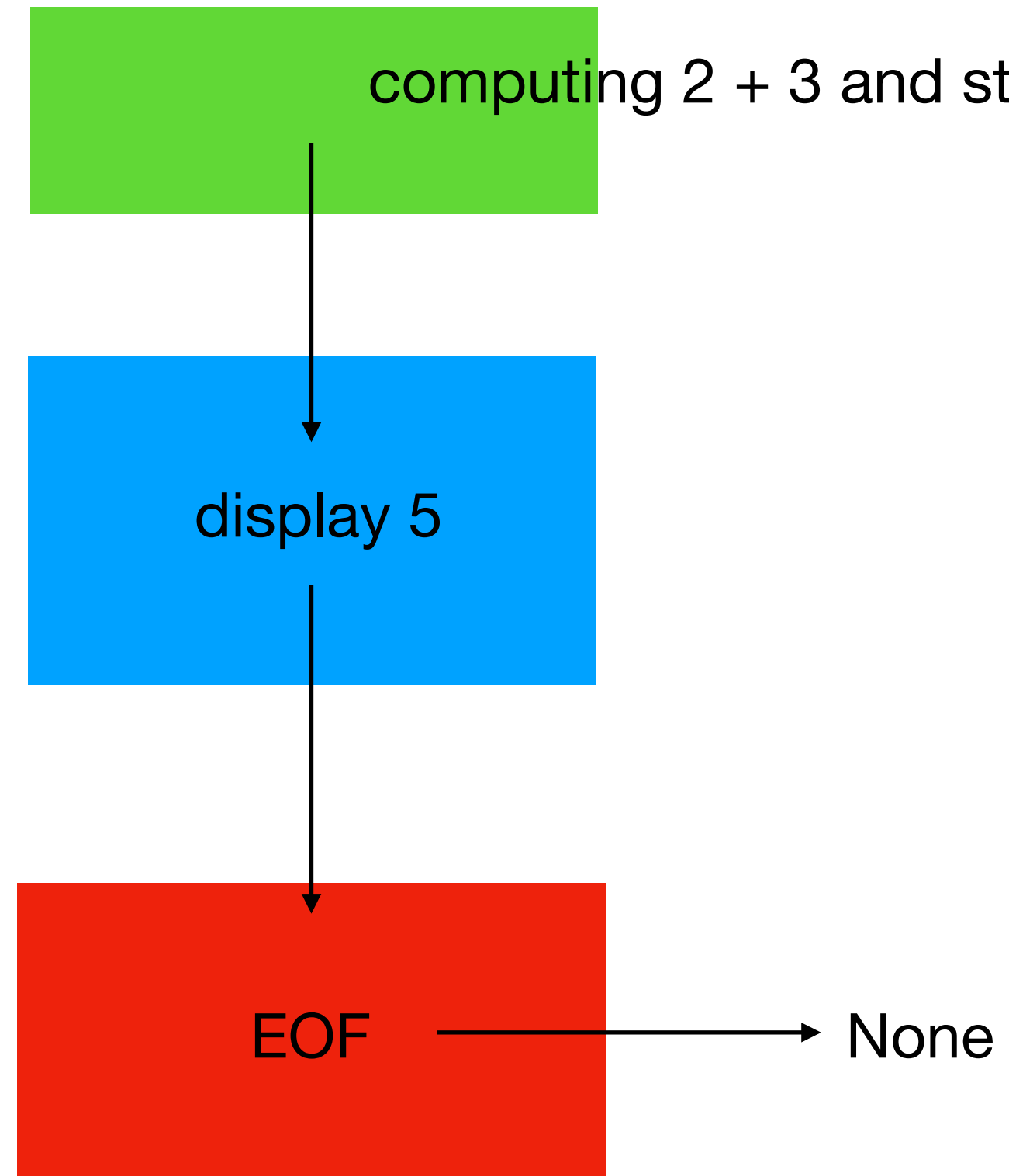


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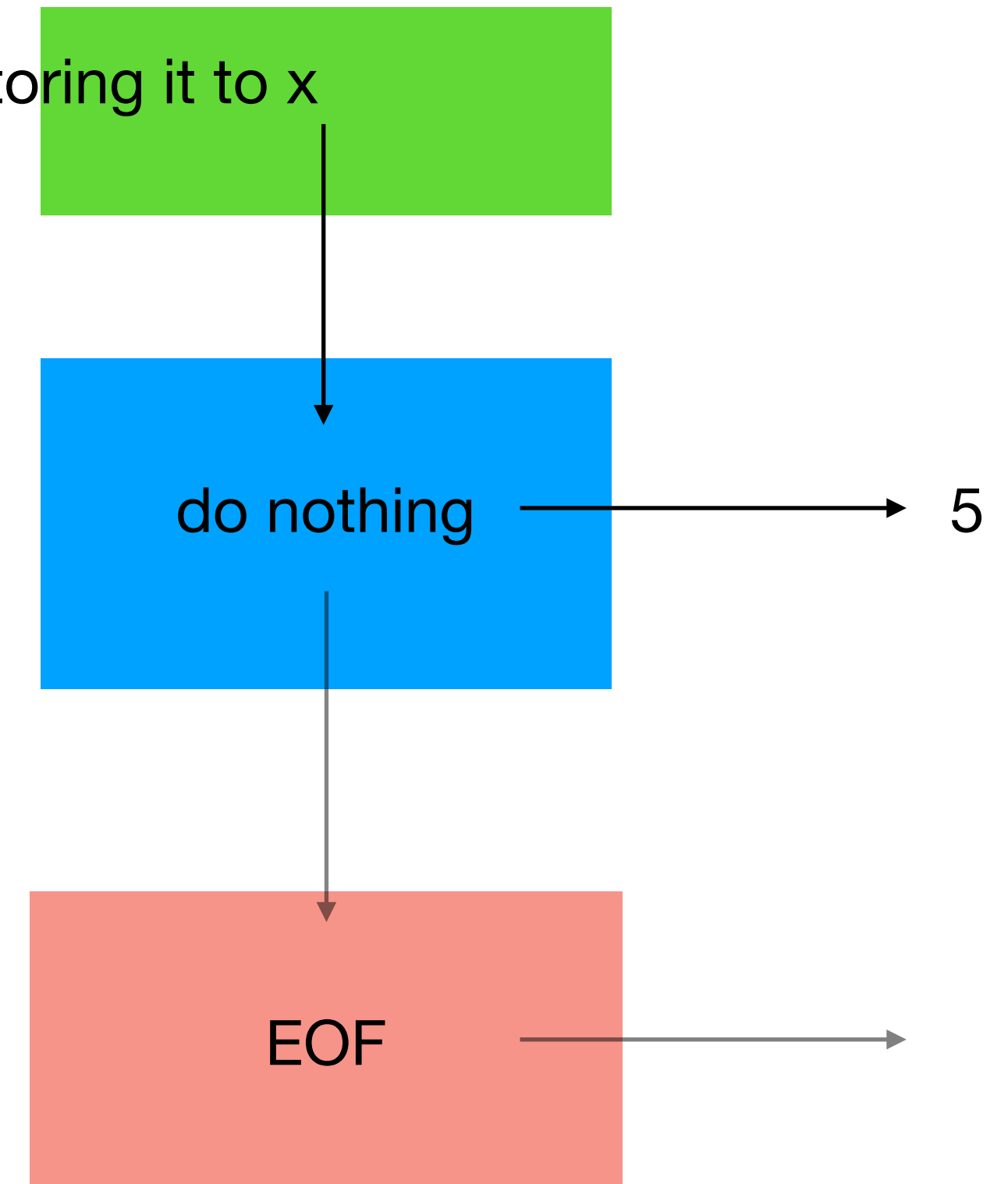
# returning

return stops procedure and outputs something  
print is an action, function

```
def showFivePrint():  
    x = 2 + 3  
    print(x)  
    return
```



```
def showFiveReturn():  
    x = 2 + 3  
    return x
```





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# returning

return stops procedure and outputs something

print is an action, function

return only 1 thing but can also return tuples (pair structures)

```
def showFivePrint():  
    x = 2 + 3  
    print(x)
```

```
> val = showFivePrint()  
5  
> val  
> val is None  
True
```

```
def showFiveReturn():  
    x = 2 + 3  
    return x
```

```
> val = showFiveReturn()  
> val  
5
```

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# function calls

```
> max(10 + 5, 9, double(18))  
36
```

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# function calls

> `max(10 + 5, 9, double(18))` → `max`  
36  
`10 + 5 = 15`  
`9 = 9`  
`double(18) = 36`  
`max(15, 9, 36)`  
`= 36`

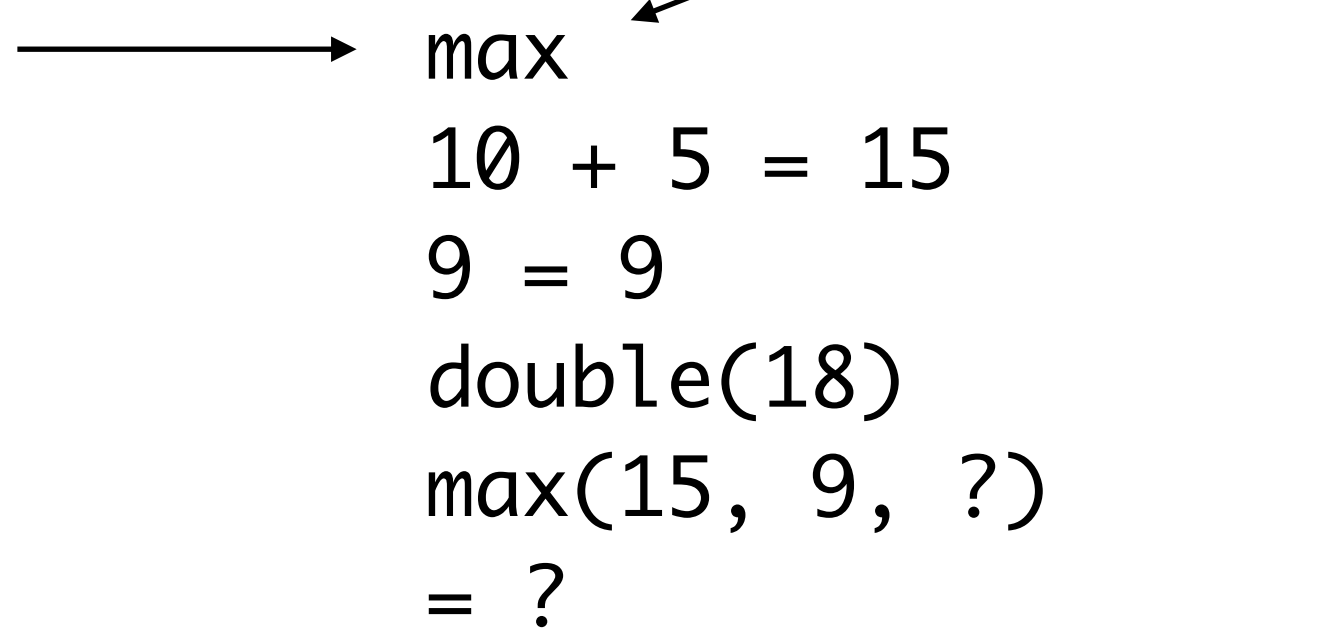
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# function calls

built-in func max(...)

> max(10 + 5, 9, double(18))  
36

→ max  
10 + 5 = 15  
9 = 9  
double(18)  
max(15, 9, ?)  
= ?



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# function calls

```
> max(10 + 5, 9, double(18))  
36
```

→ max  
10 + 5 = 15  
9 = 9  
double(18) = 36  
max(15, 9, 36)  
= 36

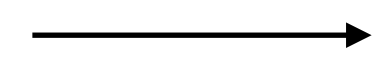
built-in func max(...)

func double(x)  
x = 18  
r.v. 36

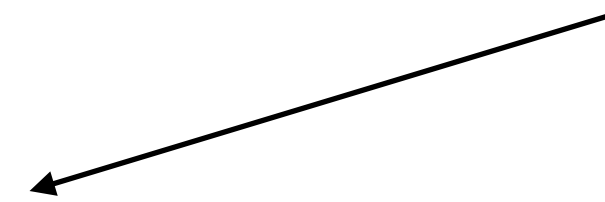
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# function calls

> max(10 + 5, 9, double(18))  
36



max  
10 + 5 = 15  
9 = 9  
double(18) = 36  
max(15, 9, 36)  
= 36



if any part of this breaks,  
you get an error and stops

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# function calls

```
> max(10 + 5, 9, 18 / 0)  
error
```

→ max  
10 + 5 = 15  
9 = 9  
18 / 0 = ?  
~~max(15, 9, 18 / 0)~~  
error

if any part of this breaks,  
you get an error and stops

typing an error != will error

EVALUATE OPERATOR

EVALUATE OPERANDS

APPLY OPERATOR



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last week...

inputs of functions

internals of functions

outputs of functions

**Higher  
Order  
Functions**

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**env. diagrams 101**

**on board**

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# env. diagrams

when do i open a frame?

what is a function's parent frame?

do we copy *intrinsically same* functions during assignment?

how do we look up variables?

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# lambdas

Lambda <arguments>: <return value>

Lambda x, y: x + y

Lambda: Lambda x: x

(Lambda x: Lambda x: x)(2)(3)

which x?

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# recursion

things *defined* by themselves

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~~recursion~~

*things defined by themselves*

**NOT ON MT1**  
**yay!**

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# recursion

factorial!

```
def factorial(n):  
    if n == 0:  
        return 1  
    else:  
        return n * factorial(n - 1)
```

$$5! = 5 * 4!$$

$$4! = 4 * 3!$$

$$3! = 3 * 2!$$

$$2! = 2 * 1!$$

$$1! = 1 * 0!$$

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# recursion

factorial!

```
def factorial(n):  
    if n == 0:  
        return 1  
    else:  
        return n * factorial(n - 1)
```

$$5! = 5 * 4!$$

$$4! = 4 * 3!$$

$$3! = 3 * 2!$$

$$2! = 2 * 1!$$

$$1! = 1 * 0!$$

uhhhhhhhhhh

**when do i stop?**



$$0! = 0 * -1!$$
$$-1! = -1 * -2! \dots$$



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# recursion

factorial!

```
def factorial(n):  
    if n == 0:  
        return 1  
    else:  
        return n * factorial(n - 1)
```

$$5! = 5 * 4!$$

$$4! = 4 * 3!$$

$$3! = 3 * 2!$$

$$2! = 2 * 1!$$

$$1! = 1 * 0!$$

uhhhhhhhhh

**base case!**

$$0! = 0 * -1!$$
$$-1! = -1 * -2! \dots$$

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# recursion

factorial!

```
def factorial(n):  
    if n == 0:  
        return 1  
    else:  
        return n * factorial(n - 1)
```

$$5! = 5 * 4!$$

$$4! = 4 * 3!$$

$$3! = 3 * 2!$$

$$2! = 2 * 1!$$

$$1! = 1 * 0!$$

yay! 🥰

**base case!**

$$0! = 1$$

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# recursion

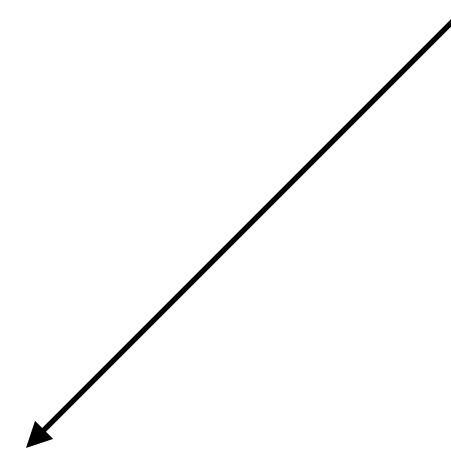
factorial!

```
def factorial(n):  
    if n == 0:  
        return 1  
    else:  
        return n * factorial(n - 1)
```

how do we come up with this

by definition, 🤖

by assuming it works, 😄



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# recursion

factorial!

```
def factorial(n):  
    if n == 0:  
        return 1  
    else:  
        return n * factorial(n - 1)
```

how do we calculate 5!

$$5! = 5 * 4!$$

for this to be true, don't we have to assume that '!' really does what it says

well in code we can't name a function '!'

we assume that (n-1)! works

recursive leap of faith

well... i have to test it by tracing it

well... big headache

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# recursion

strategy

if you capture all the base cases  
you can assume it works

so you can create the recursive call

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# recursion

*motivation for it*

input

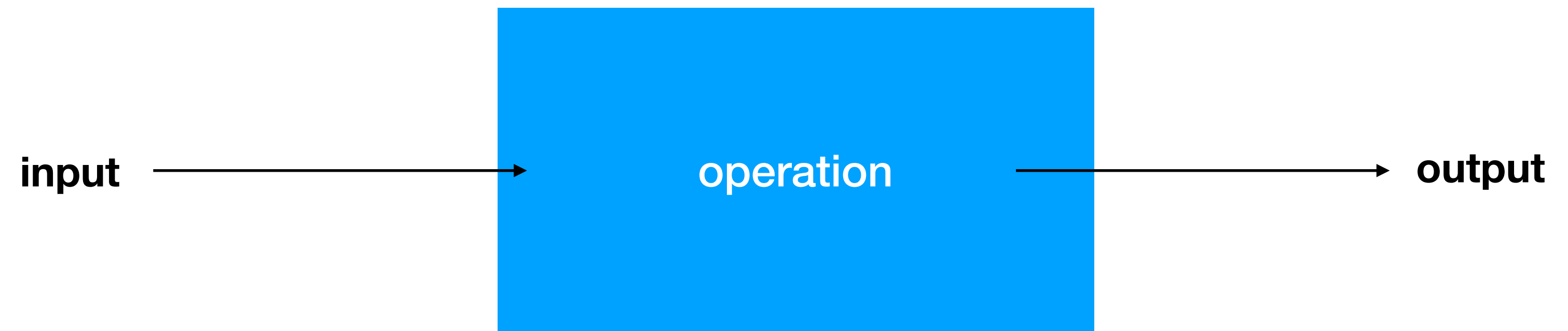


operation

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# recursion

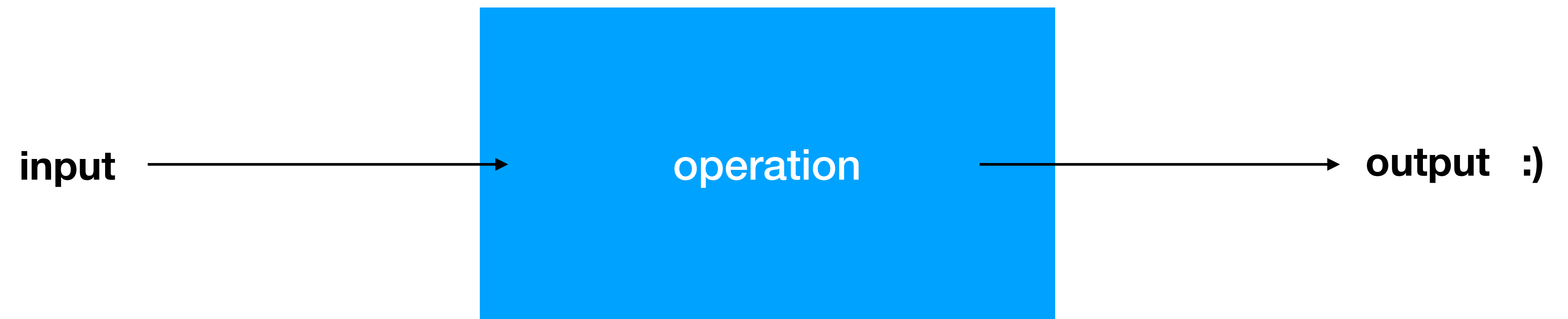
things *defined* by themselves



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# recursion

things *defined* by themselves

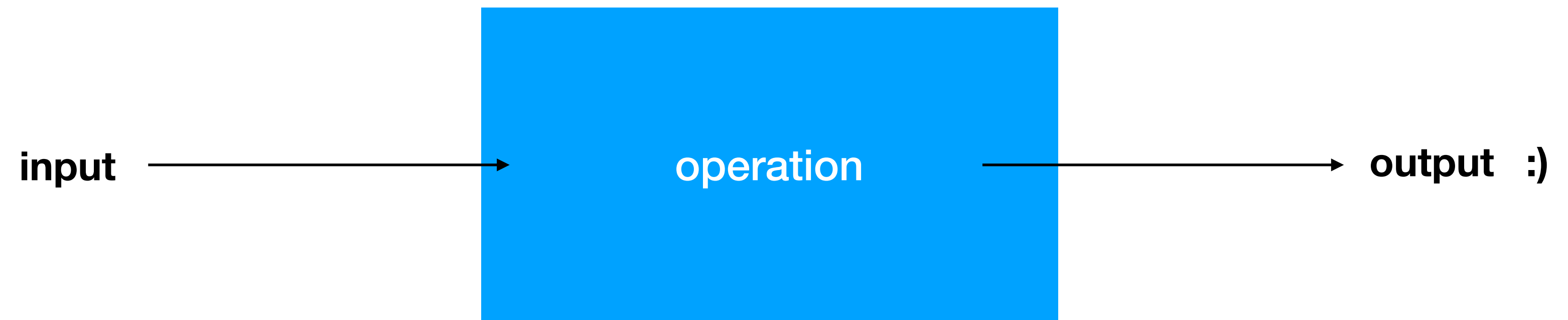




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# recursion

things *defined* by themselves



tell me the number of ways to line \$26

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# recursion

things *defined* by themselves

?

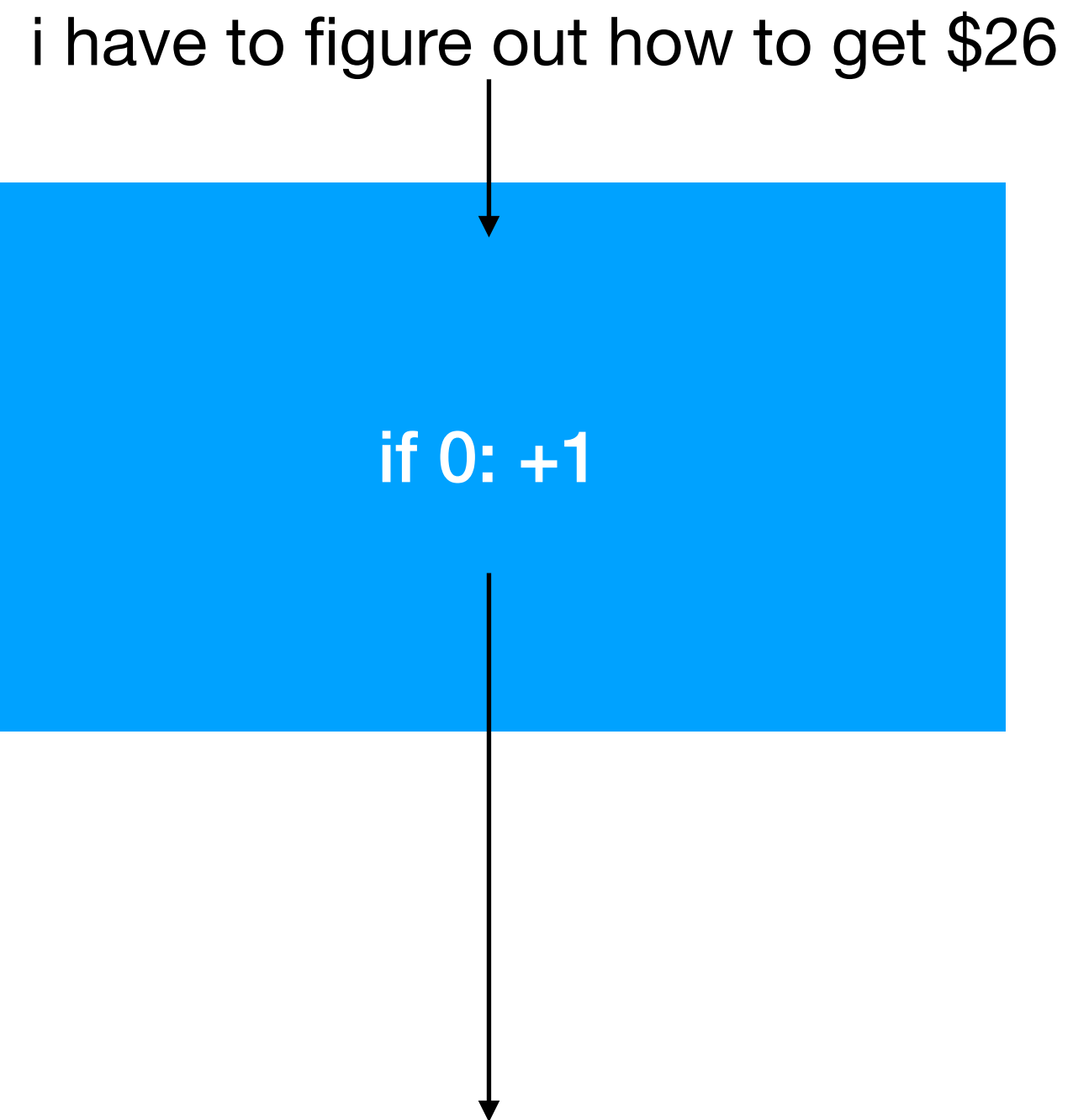
input = 26



operation

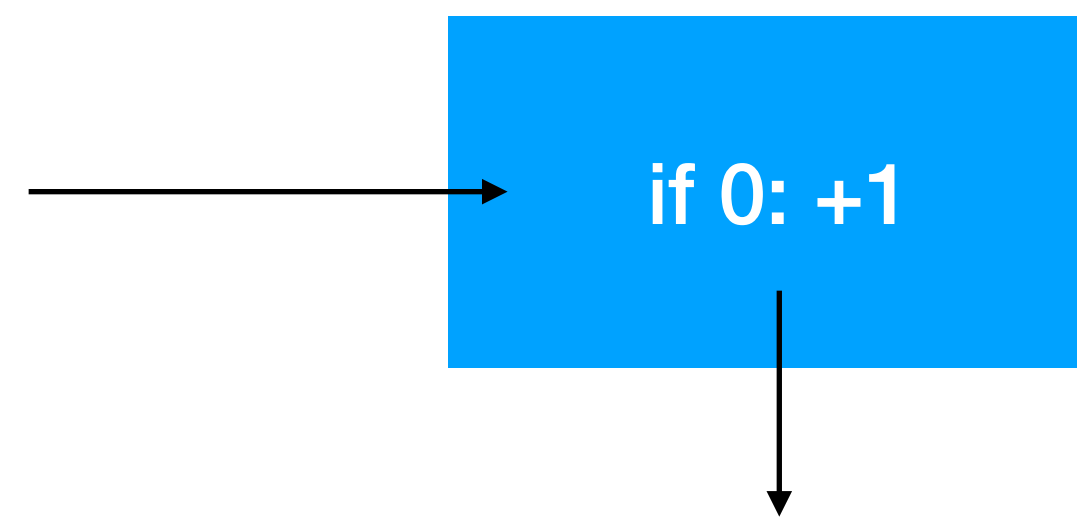
count :)

tell me the number of  
ways to line \$26

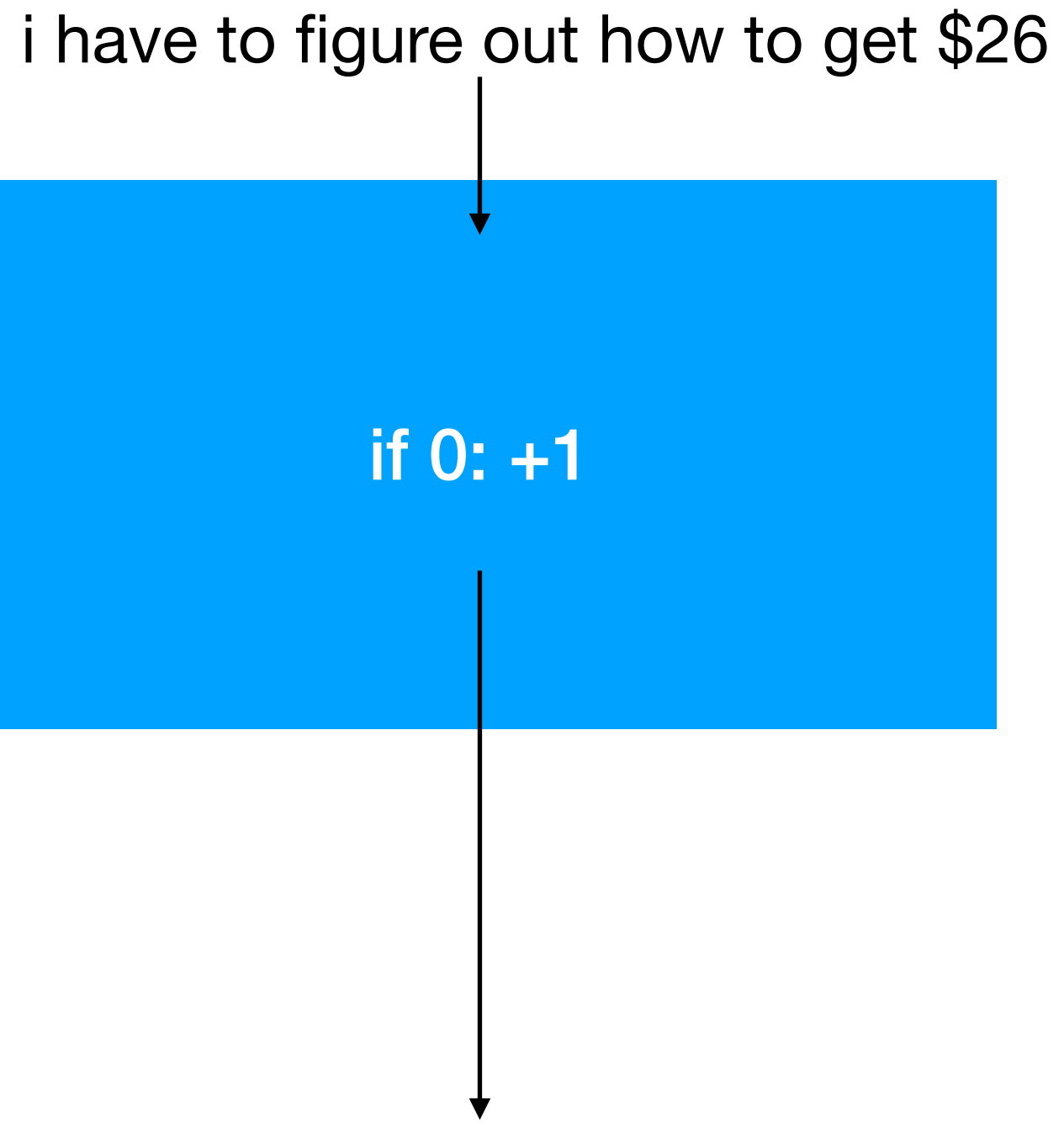


the blue boxes are operations!

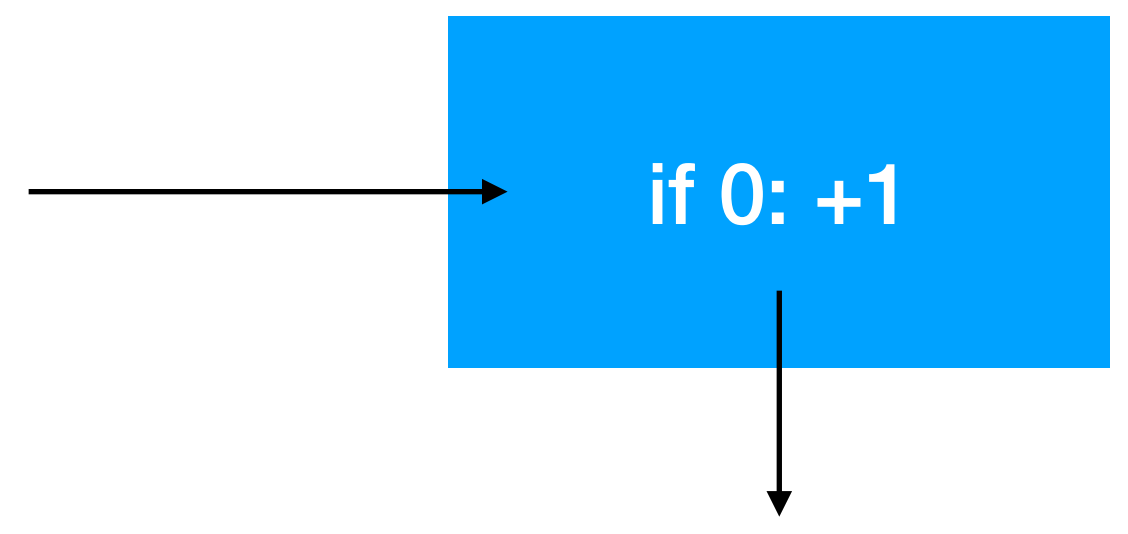
if i use \$1 as my first denomination  
i have to figure out how to get the \$25  
·  
·  
·  
if i use \$20 as my first denomination  
i have to figure out how to get the \$6



if i use \$1 as my first denomination  
i have to figure out how to get the \$24  
·  
·  
·

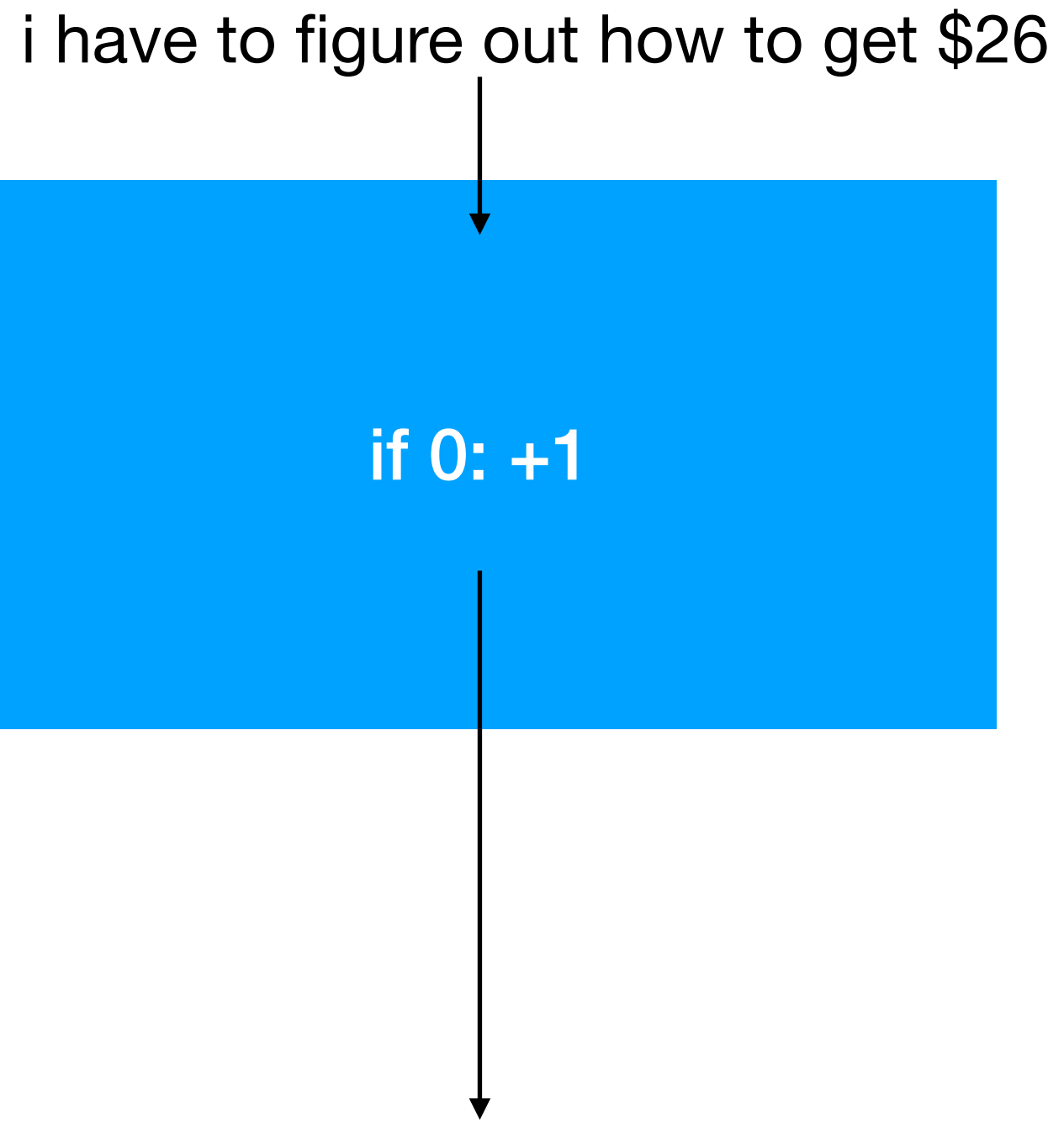


if i use \$1 as my first denomination  
i have to figure out how to get the \$25  
·  
·  
·  
if i use \$20 as my first denomination  
i have to figure out how to get the \$6

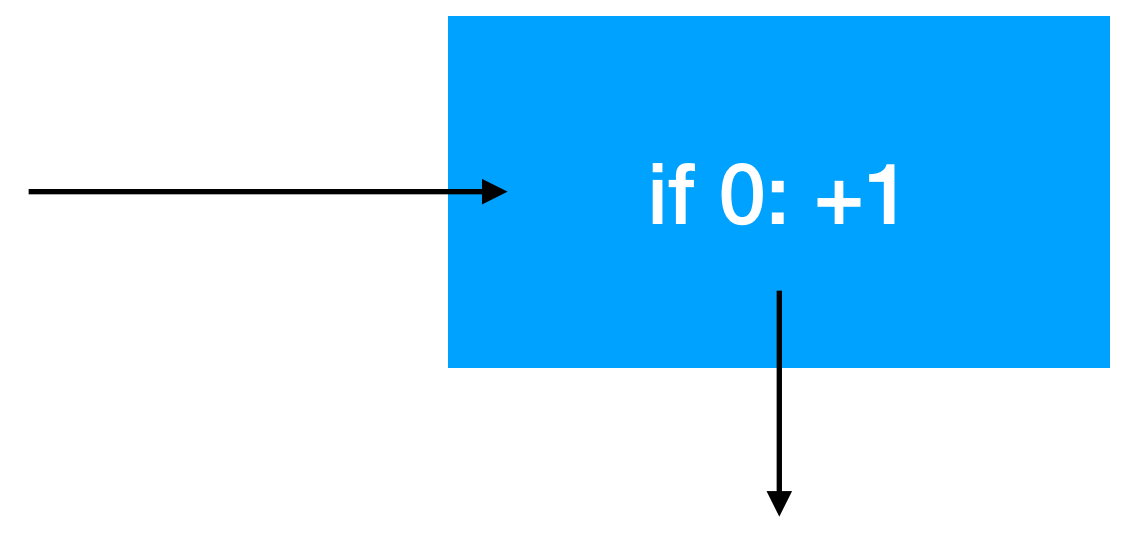


if i use \$1 as my first denomination  
i have to figure out how to get the \$24  
·  
·  
·

—————→ **output**



if i use \$1 as my first denomination  
i have to figure out how to get the \$25  
·  
·  
·  
if i use \$20 as my first denomination  
i have to figure out how to get the \$6



if i use \$1 as my first denomination  
i have to figure out how to get the \$24  
·  
·  
·

—————→ **output :(**

```
def count(n):
    total = 0
    options = [n]
    while len(options) > 0:
        curr = options.pop(0)
        for change in [1, 5, 10, 20]:
            val = curr - change
            if val == 0:
                total += 1
            elif val > 0:
                options.append(val)
    return total
```

```
def count(n):
    total = 0
    options = [n]
    while len(options) > 0:
        curr = options.pop(0)
        for change in [1, 5, 10, 20]:
            val = curr - change
            if val == 0:
                total += 1
            elif val > 0:
                options.append(val)
    return total
```



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# recursion

things *defined* by themselves

?



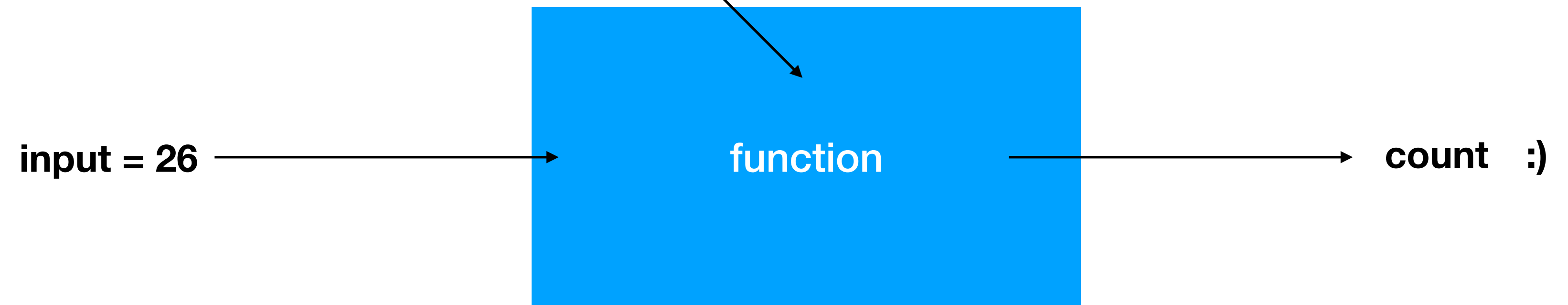


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# recursion

things *defined* by themselves

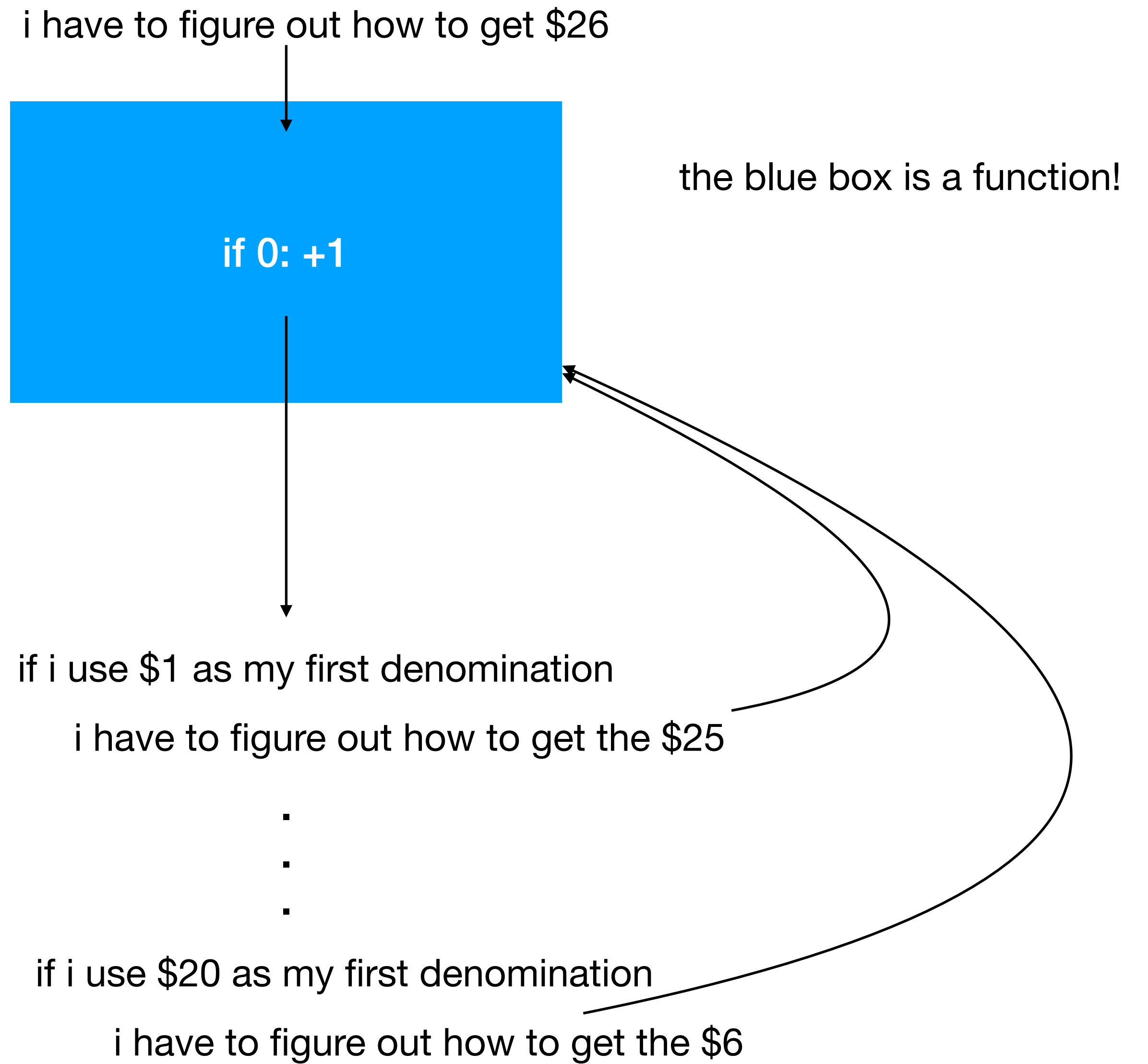
?



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# recursion

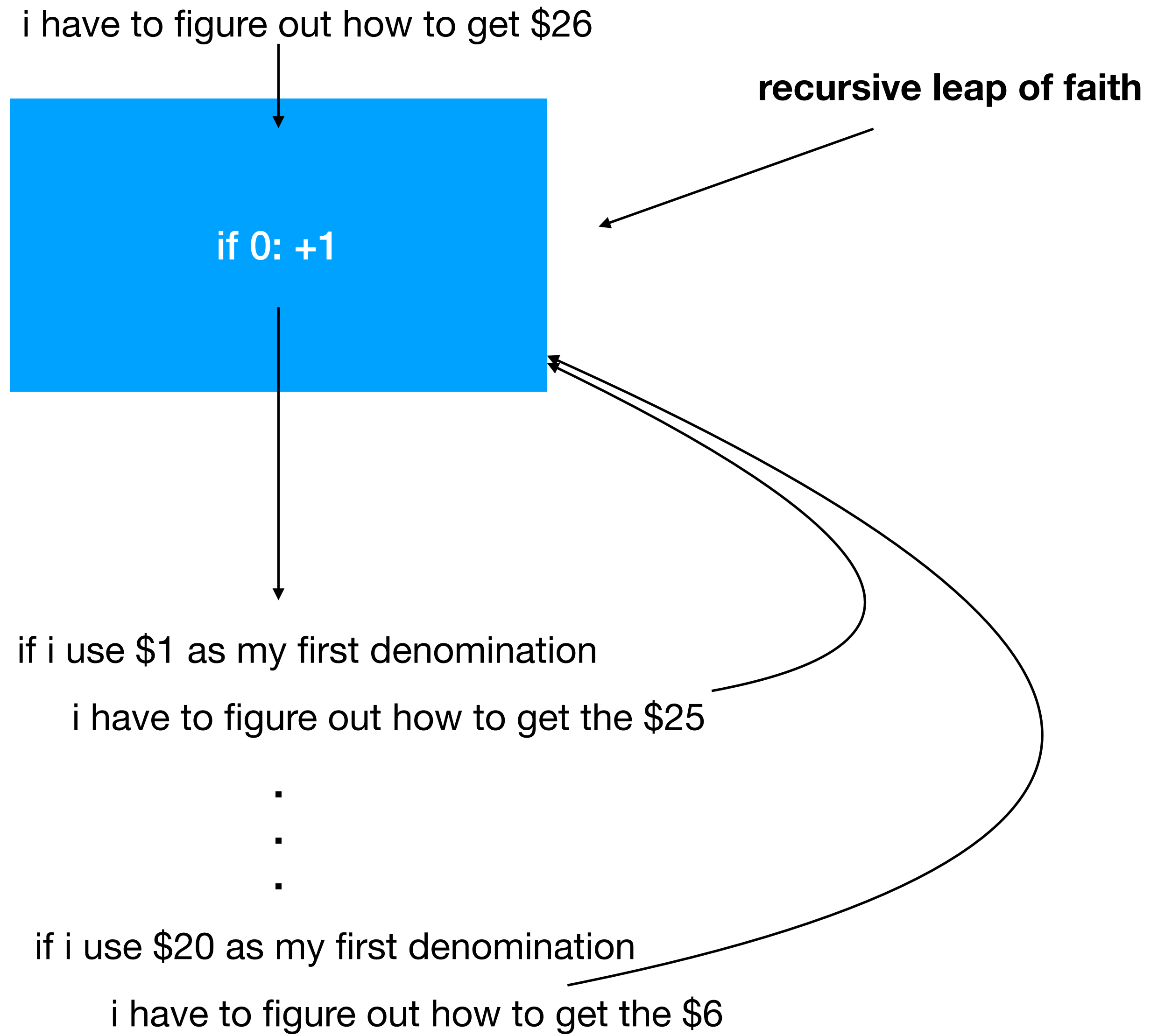
things *defined* by themselves



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# recursion

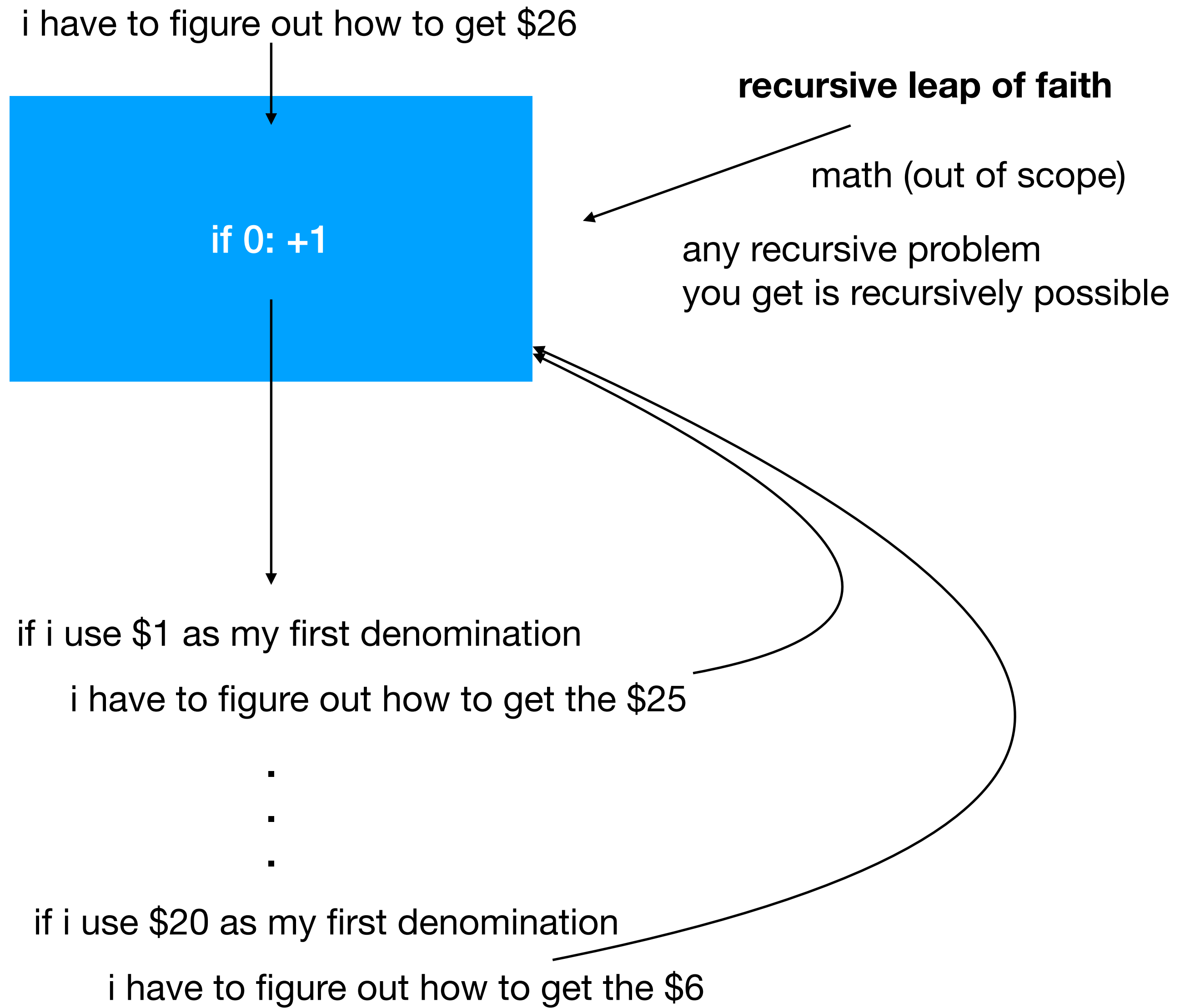
things *defined* by themselves



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# recursion

things *defined* by themselves

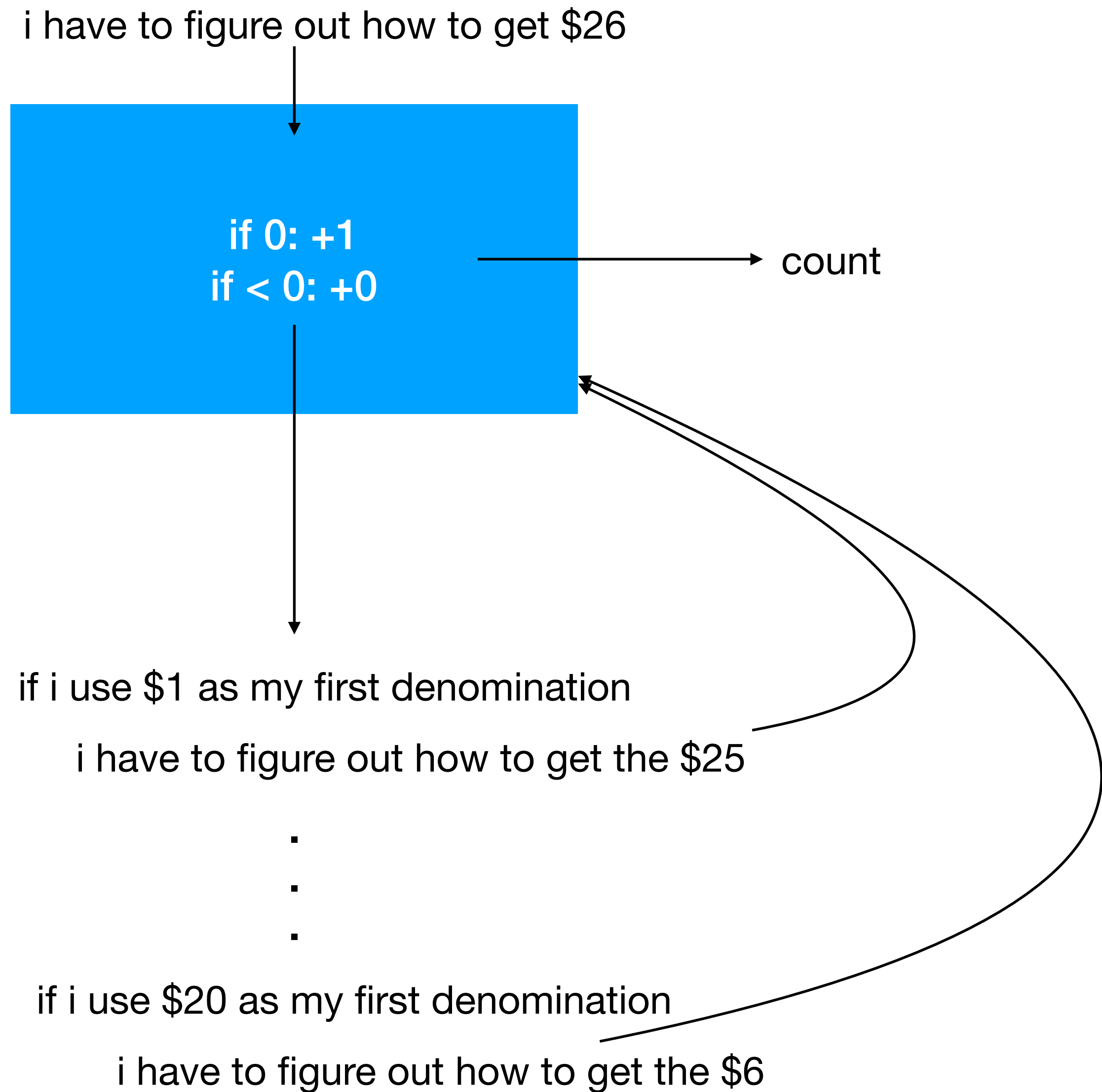


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# recursion

things *defined* by themselves

```
def count_recurse(n):  
    if n < 0:  
        return 0  
    elif n == 0:  
        return 1  
    else:  
        return count_recurse(n - 1)  
            + count_recurse(n - 5)  
            + count_recurse(n - 10)  
            + count_recurse(n - 20)
```



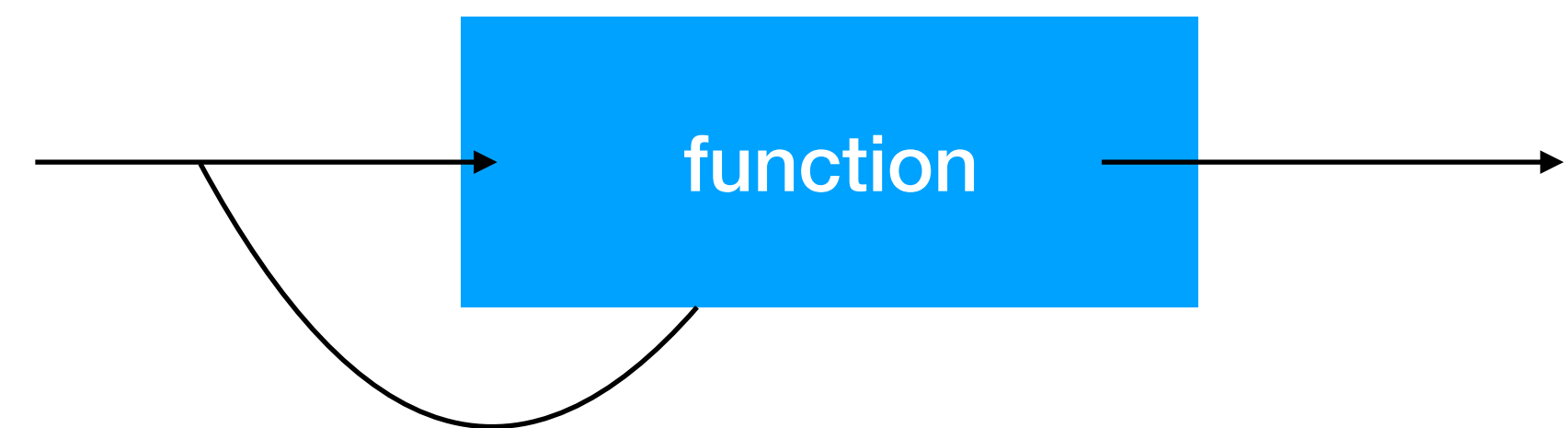
```
def count(n):
    total = 0
    options = [n]
    while len(options) > 0:
        curr = options.pop(0)
        for change in [1, 5, 10, 20]:
            val = curr - change
            if val == 0:
                total += 1
            elif val > 0:
                options.append(val)
    return total
```

```
def count_recurse(n):
    if n < 0:
        return 0
    elif n == 0:
        return 1
    else:
        return count_recurse(n - 1)
            + count_recurse(n - 5)
            + count_recurse(n - 10)
            + count_recurse(n - 20)
```

```
def count(n):
    total = 0
    options = [n]
    while len(options) > 0:
        curr = options.pop(0)
        for change in [1, 5, 10, 20]:
            val = curr - change
            if val == 0:
                total += 1
            elif val > 0:
                options.append(val)
    return total
```



```
def count_recurse(n):
    if n < 0:
        return 0
    elif n == 0:
        return 1
    else:
        return count_recurse(n - 1)
        + count_recurse(n - 5)
        + count_recurse(n - 10)
        + count_recurse(n - 20)
```



```
def count_recursion(n):
    to be optimized
    while n > 0:
        count_recursion(n - 1)
    for coin in [1, 5, 10, 20]:
        change = n - coin
        if change >= 0:
            count_recursion(change)
    return 1
```

```
def count_recurse(n):
    if n < 0:
        return 0
    elif n == 0:
        return 1
    else:
        return count_recurse(n - 1)
            + count_recurse(n - 5)
            + count_recurse(n - 10)
            + count_recurse(n - 20)
```





example too complex... for now

**so what does this mean**

we have a strategy on how to create recursive functions

we can see that recursion isn't pointless...

at least for more complex problems

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# midterm 1

10% of your grade

there's extra credit on every project

epa points

CS61A

# midterm 1

10% of your grade

there's extra credit on every project

epa points

*one test isn't going to determine*

*if you should be a computer scientist*